Transverse Symmetry and Spin-$3/2$ Fields

D. Blas[a1],

[a]*Departament de Física Fonamental & Institut de Ciències del Cosmos,*
Universitat de Barcelona, Diagonal 647, 08028 Barcelona, Spain.

Abstract

We study the possible covariant Lagrangians that describe the propagation of pure spin-3/2 particles. We show that, apart from the well-known Rarita-Schwinger Lagrangian, there is another possibility where the field is described by a γ-traceless combination and that both Lagrangians yield the same physical predictions for the interaction of conserved sources. We also prove that for the case when the spin-2 field is described by a traceless field, there is no possible spin-3/2 action that makes the system supersymmetric. Nevertheless, the interaction between this field and the spin-3/2 field may be possible.

1   Introduction

It has been recently shown that the free massless spin-2 field can be consistently described by a Lagrangian depending on the traceless tensor field

$$\hat{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{4}\eta_{\mu\nu}h, \tag{1}$$

and endowed with a *reducible* gauge symmetry [1, 2]. This formulation is in some sense opposite to the standard approach of higher spin, which resorts to the introduction of auxiliary fields to build a covariant Lagrangian which yields the correct equations of motion [3, 4, 5, 6] (see also [7, 8] for the massive case). The analysis has been extended to bosonic fields of higher spin in [9]. In general, even if the Lagrangians for the traceless field are related to the original Fronsdal model of [4], the equivalence between both formulations is not trivial. In fact, a naive counting indicates that the Lagrangian of the traceless field has an extra propagating degree of freedom. However, as shown in [9], the appearance of a *tertiary* constraint in this description kills the extra degree of freedom and makes both theories equivalent at the classical level (except for an integration constant). This phenomenon is reminiscent of the reason why *unimodular* gravity is equivalent to General Relativity [10].

The covariant description of *fermionic* fields of spin $s > 1/2$ also needs the introduction of auxiliary fields which are rendered spurious by an associated gauge symmetry [5]. A natural question one may ask is whether, as happens in the bosonic case, there exists more than one Lorentz invariant Lagrangian that describes the propagation of the higher spin particles. Besides, from the analogy with the bosonic case, one may expect that the new Lagrangian depends on a field with fewer degrees of freedom than in the standard formulation. One of the purposes of this work is to study this possibility for the spin-3/2 field.

Besides, it is well known that the standard spin-2 and spin-3/2 actions constitute a supersymmetric action [11]. This property is crucial in finding an action which couples the spin-3/2 field *consistently*[2] to the graviton and we will look for a similar minimal supersymmetric extension for the traceless case.

The paper is organized as follows. In Section 2, we will present the most general Lorentz invariant Lagrangian for the Majorana spin-3/2 in terms of the spinor-vector ψ_μ, and identify the possible gauge symmetries of the action. In Section 3, we will study the propagators of the Lagrangians which are endowed with a gauge symmetry and comment on the possibility of consistently coupling the field to the electromagnetic field. In Section 4, we will study the possible supersymmetric linear actions for the traceless spin-2 field. We will finally present our conclusions and outlook.

We will follow the conventions of [12][3] and work with a Majorana vector-spinor ψ_μ.

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[2] By consistent coupling we mean that the coupled theory is invariant under a deformation of the gauge symmetry of the free theory. This is understood as a necessary condition for consistency as the gauge symmetry is essential to make the theory *unitary*.

[3] We will work with

$$\eta_{\mu\nu} = \{+, -, -, -\}, \quad \{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}, \quad \gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3, \quad \epsilon^{0123} = 1. \tag{2}$$

2 Lagrangians for Pure Massless Spin-$3/2$

The most general local, Lorentz invariant Lagrangian for a massless Majorana vector-spinor ψ_μ and involving just first order derivatives is given by[4]

$$\mathcal{S}^{(3/2)} = \int \mathrm{d}^4x \; \bar{\psi}_\mu \left(\lambda(\gamma^\mu\partial^\nu + \gamma^\nu\partial^\mu) + \vartheta\gamma^\mu\slashed{\partial}\gamma^\nu + \zeta\eta^{\mu\nu}\slashed{\partial} \right) \psi_\nu. \tag{3}$$

After a transformation of the form

$$\psi_\mu \mapsto \psi_\mu - \frac{a}{4}\gamma_\mu\gamma^\rho\psi_\rho, \tag{4}$$

the coefficients change as

$$\lambda \mapsto \lambda\,(1-a) - \frac{a}{2}\zeta, \quad \vartheta \mapsto \vartheta(1-a)^2 - \frac{a(1-a)}{2}\lambda + \frac{a}{2}\left(1 - \frac{a}{4}\right)\zeta. \tag{5}$$

This transformation is a field redefinition which makes one of the coefficients spurious except for the case $a = 1$. In this pathological case, the transformation is *not* invertible.

The Majorana field ψ_μ has 16 real independent components, all of which will be dynamical for a general action of the form (3). However, if the action is to describe *just* a massless particle of spin-3/2, only the $\pm 3/2$ polarizations should be dynamical, which implies the need for a gauge symmetry to render the remaining polarizations non-dynamical[5]. In fact, the standard action for spin-3/2, known as Rarita-Schwinger (RS) and characterized by the values $\lambda = -\vartheta = -\zeta$ [13] (and the coefficients related to it by a transformation (5) for $a \neq 1$), is invariant under the transformation

$$\psi_\mu \mapsto \psi_\mu + \partial_\mu\epsilon, \tag{6}$$

where ϵ is a Majorana spinor. This transformation can be generalized to

$$\psi_\mu \mapsto \psi_\mu + \partial_\mu\epsilon + \gamma_\mu\varphi, \tag{7}$$

which is the most general covariant gauge transformation for the field ψ_μ which does not involve the spin-3/2 components of the field. Under these transformations, the general action (3) changes as

$$\delta S^{(3/2)} = -2 \int \mathrm{d}^4x \Big(\{(\lambda+\vartheta)\Box\bar{\epsilon} + (\lambda + 4\vartheta - \zeta)\partial^\alpha\bar{\varphi}\gamma_\alpha\}\gamma^\mu\psi_\mu$$

$$-\{(\lambda+\zeta)\partial^\alpha\bar{\epsilon}\gamma_\alpha + 2(2\lambda+\zeta)\bar{\varphi}\}\partial^\mu\psi_\mu \Big). \tag{8}$$

For $2\lambda + \zeta \neq 0$ the previous variation cancels for

$$\bar{\varphi} = -\frac{(\lambda+\zeta)\partial^\alpha\bar{\epsilon}\gamma_\alpha}{2(2\lambda+\zeta)}, \quad (3\lambda^2 + 2\zeta\lambda + \zeta^2 - 2\vartheta\zeta)\Box\bar{\epsilon} = 0. \tag{9}$$

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[4]For a Dirac spinor, the coefficients in front of the first and second terms do not necessarily coincide.

[5]Recall also that fermions have half as many propagating degrees of freedom as components, since the other half are canonical momenta.

In other words, for

$$\vartheta = \frac{\zeta^2 + 2\zeta\lambda + 3\lambda^2}{2\zeta}, \quad 2\lambda + \zeta \neq 0, \tag{10}$$

the action (3) is invariant under (7) with

$$\bar{\varphi} = -\frac{(\lambda + \zeta)\partial^\alpha \bar{\epsilon}\gamma_\alpha}{2(2\lambda + \zeta)},$$

and ϵ remains a *free* Majorana spinor. As it is clear from (5), not all these possibilities are independent. In fact, they are *all* equivalent to the RS action after a field redefinition with parameter

$$a = \frac{2(\lambda + \zeta)}{\zeta}.$$

For the singular case $2\lambda + \zeta = 0$, the variation of the action cancels provided that

$$\partial\!\!\!/\epsilon = 0, \quad (\lambda + 4\vartheta - \zeta)\partial^\alpha \bar{\varphi}\gamma_a = 0. \tag{11}$$

In this case, the Majorana spinor φ will be free if $\lambda = \zeta - 4\vartheta$, which, together with $2\lambda + \zeta = 0$, implies that

$$\lambda = -\frac{1}{2}\zeta, \quad \vartheta = \frac{3}{8}\zeta. \tag{12}$$

Substituting the previous values in (3) (and fixing ζ), one finds the action

$$\mathcal{S}_{\mathrm{WRS}}^{(3/2)} \equiv \mathcal{S}_{\mathrm{RS}}^{(3/2)}(\hat{\psi}_\mu) = -\frac{1}{2}\int \mathrm{d}^4x \; \bar{\hat{\psi}}_\mu \epsilon^{\mu\nu\rho\sigma}\gamma_5\gamma_\nu\partial_\rho\hat{\psi}_\sigma, \tag{13}$$

where $\mathcal{S}_{\mathrm{RS}}^{(3/2)}$ refers to the RS action and $\hat{\psi}_\mu \equiv \psi_\mu - \frac{1}{4}\gamma_\mu\gamma^\alpha\psi_\alpha$. Thus, this action corresponds to the singular transformation (4) with $a = 1$. This means that the action is written in terms of a covariant quantity with *fewer* degrees of freedom than the original field ψ_μ. Indeed, $\hat{\psi}_\mu$ is γ-traceless,

$$\gamma^\mu\hat{\psi}_\mu = 0,$$

which means that $\hat{\psi}_\mu$ has 12 independent real components. The WRS label comes from the analogy of the transformation (7) involving the field φ (known as special supersymmetry, or simply, S-symmetry [14]) with the Weyl gauge symmetry. Besides, the first condition in (10) is also satisfied in this case.

As happens in the spin-2 case, for generic ϵ and φ there is no action in the family (3) invariant under the most general possible gauge symmetry[6]. In particular, this means that some of the low spin components of the field ψ_μ may be dynamical as they are not automatically killed by the gauge symmetry. To our knowledge, the action (13) has not been studied in detail in the past (for the RS action see, *e.g.*, [11, 12]).

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[6]Similarly to what happens to the Weyl symmetry for theories invariant under diffeomorphisms, an action invariant under this gauge group exists once higher derivatives terms are included, but the theory is not unitary (see [14]).

In terms of $\hat{\psi}_\mu$, the action is invariant under the transformation

$$\delta\hat{\psi}_\mu = \partial_\mu\epsilon, \tag{14}$$

satisfying $\partial\!\!\!/\epsilon = 0$. If we, instead, consider ψ_μ as the fundamental field and think of $\hat{\psi}_\mu$ as a derived quantity, the action is invariant under (7) with $\partial\!\!\!/\epsilon = 0$. It is important to note that the action (13) is *not* related to the RS action by a gauge fixing term, as the only covariant gauge fixing term just involves the term proportional to ϑ in (3).

There is another way in which we can show that the RS and the WRS actions are the only possibilities out of the general action (3) endowed with a gauge symmetry. Namely, we can analyze the structure of the equations of motion of the theory and find all the degenerate possibilities. To this aim, it is convenient to decompose the field ψ_μ into irreducible representations of the $SO(3)$ group (see *e.g.* [15]),

$$\psi_0 = A, \quad \psi_i = t_i + \gamma_i\chi + \partial_i E, \tag{15}$$

with $\gamma_i t^i = \partial_i t^i = 0$. The presence of the γ_i matrices in the definition of χ implies that it is an anti-Majorana fermion[7]

$$\bar{\chi} = -\chi^T C.$$

In terms of the previous fields, the general Lagrangian (3) can be decomposed into a spin-3/2 and a spin-1/2 part,

$$\mathcal{L} = \mathcal{L}^{(3/2)} + \mathcal{L}^{(1/2)}, \tag{16}$$

with $\mathcal{L}^{(3/2)} \equiv -\zeta\bar{t}_i\partial\!\!\!/\,t_i$ and

$$\mathcal{L}^{(1/2)} \equiv \bar{E}\left\{(\zeta - \vartheta)\gamma_0\partial_0 - (2\lambda + \vartheta + \zeta)\gamma_i\partial_i\right\}\Delta E + \bar{A}\left\{(2\lambda + \vartheta + \zeta)\gamma_0\partial_0 - \gamma_i\partial_i(\zeta - \vartheta)\right\}A$$
$$+\bar{\chi}\left\{3(3\vartheta - \zeta)\gamma_0\partial_0 - \gamma_i\partial_i(6\lambda + 9\vartheta - \zeta)\right\}\chi + 2\bar{\chi}\left\{-(4\lambda + 3\vartheta + \zeta)\Delta E - (3\vartheta - \zeta)\gamma_0\gamma_i\partial_0\partial_i E\right\}$$
$$+2\bar{A}\left\{-(\lambda + 3\vartheta)\gamma_0\gamma_i\partial_i\chi + (\lambda + \vartheta)[\partial_0(3\chi - \gamma_i\partial_i E) - \gamma_0\Delta E]\right\}.$$

The presence of a gauge symmetry can be identified by the singular character of the kinetic term of the equations of motion [16]. The kinetic term can be written as

$$\begin{pmatrix} (\zeta - \vartheta)\gamma_0\Delta & (\zeta - 3\vartheta)\gamma_0\gamma_i\partial_i & (\lambda + \vartheta)\gamma_i\partial_i \\ (\zeta - 3\vartheta)\gamma_0\gamma_i\partial_i & 3(3\vartheta - \zeta)\gamma_0 & 3(\lambda + \vartheta) \\ -(\lambda + \vartheta)\gamma_i\partial_i & 3(\lambda + \vartheta) & (2\lambda + \vartheta + \zeta)\gamma_0 \end{pmatrix} \begin{pmatrix} \dot{E} \\ \dot{\chi} \\ \dot{A} \end{pmatrix}.$$

The determinant of the matrix multiplying the time derivative of the fields is

$$16\zeta^4(-2\vartheta\zeta + \zeta^2 + 2\zeta\lambda + 3\lambda^2)^4\Delta^4. \tag{17}$$

Thus, we find that the theory will include constraints whenever (we take $\zeta \neq 0$ as otherwise the spin-3/2 degrees of freedom are not present)

$$\vartheta = \frac{\zeta^2 + 2\zeta\lambda + 3\lambda^2}{2\zeta}. \tag{18}$$

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[7]We can also define $\chi = \gamma_0\eta$ with η being a Majorana spinor.

As we found previously, this condition corresponds to the existence of a gauge symmetry of the form (7). The previous decomposition can be used to identify the Lagrange multipliers and constraints of the theory. Once the constraints are introduced back into the action (16), the only remaining fields are the propagating degrees of freedom.

3 Propagator and Coupling of the WRS Lagrangian

The analysis of the general Lagrangian (3) is beyond the scope of this work. The absence of a gauge symmetry means that all the degrees of freedom propagate in general and one expects the presence of ghosts in the low-spin states [11]. For the RS Lagrangian, the propagator, spin content and unitarity properties can be found in [17, 18, 11]. In this case, the fact of having a gauge symmetry involving a derivative allows to kill all the low-spin states, leaving just the $\pm 3/2$ components [17].

For the WRS action the naive counting of propagating degrees of freedom implies the presence of spin-1/2 components. Let us show that this is indeed the case. From the action (13), one readily finds that the equations of motion for ψ_μ are the γ-traceless part of the RS case in the gauge $\gamma^\mu \psi_\mu = 0$. This gauge can be easily reached in both the WRS and RS case. One finds,

$$\mathcal{R}^\mu_{\mathrm{WRS}} \equiv \frac{\delta \mathcal{L}_{\mathrm{WRS}}}{\delta \bar{\psi}_\mu} = \left(\delta^\mu_\alpha - \frac{1}{4}\gamma^\mu \gamma_\alpha \right) \frac{\delta \mathcal{L}_{\mathrm{RS}}(\hat{\psi}_\mu)}{\delta \bar{\hat{\psi}}_\mu} \equiv \left(\delta^\mu_\alpha - \frac{1}{4}\gamma^\mu \gamma_\alpha \right) \mathcal{R}^\alpha_{\mathrm{RS}}(\hat{\psi}_\mu) = 0, \tag{19}$$

with $\gamma_\alpha \mathcal{R}^\alpha_{\mathrm{WRS}} = 0$, which is the Bianchi identity associated to the S-symmetry. Contracting the equations of motion with the derivative operator, one finds

$$\partial_\mu \mathcal{R}^\mu_{\mathrm{WRS}} = -\frac{1}{4}\slashed{\partial} \left(\gamma_\alpha \mathcal{R}^\alpha_{\mathrm{RS}}(\hat{\psi}_\mu) \right). \tag{20}$$

Thus, contrary to what happens in the bosonic case (cf.[1]), we do not recover the missing equations of the RS Lagrangian (in this case the γ-trace of the RS equations of motion). This result was expected as in the case under study there is no gauge invariance left in the WRS action for the $\hat{\psi}_\mu$ field, which means that no secondary or tertiary constraints can appear. In fact, from the identity

$$\gamma_\alpha \mathcal{R}^\alpha_{\mathrm{RS}}(\hat{\psi}_\mu) = -2\partial^\alpha \hat{\psi}_\alpha,$$

we see that there is a spin-1/2 propagating degree of freedom as the equation of motion for $\partial^\alpha \hat{\psi}_\alpha$ is

$$\slashed{\partial}\partial^\alpha \hat{\psi}_\alpha = 0, \tag{21}$$

in contrast to the RS case where $\partial^\alpha \hat{\psi}_\alpha$ cancels on shell[8]. Besides, the residual gauge transformation leaves this combination invariant as

$$\delta \partial^\alpha \hat{\psi}_\alpha = \Box \epsilon = 0. \tag{23}$$

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[8] These equation of motion are also obtained if we add a gauge fixing term

$$\lambda \bar{\psi}_\mu \gamma^\mu \gamma^\nu \psi_\nu \tag{22}$$

to the RS action. This is reminiscent to what happens in unimodular gravity [10].

The previous arguments imply that the free WRS case is not in general equivalent to the RS case as there is one more spin-1/2 propagating degree of freedom. However, notice that if we fix the initial condition

$$\partial^\alpha \hat{\psi}_\alpha|_0 = 0, \tag{24}$$

the equation (21) implies that the missing equations also hold and that both systems are equivalent. This situation is analogous to what happens in gauge invariant theories when one fixes the gauge through a covariant quadratic gauge fixing term [18, 19]. Thus, we conclude that both actions have the same equations of motion once we impose the (24) initial condition in the WRS case.

To show that both theories yield equivalent physical results, we can couple the free spin-3/2 field to a conserved source J_α, with $\partial^\alpha J_\alpha = 0$, and find the propagator that mediates the interaction between two sources. The most general covariant non-derivative coupling will be of the form

$$\mathcal{S}_{int} = \int \mathrm{d}^4 x \bar{\psi}_\mu \left(J^\mu - \frac{b}{4} \gamma^\mu \gamma_\alpha J^\alpha \right) + h.c.$$

The consistency of the equations of motion implies that for the RS case $b = 0$, whereas for WRS $b = 1$. The equations of motion for the WRS case are

$$\left(\delta_\alpha^\mu - \frac{1}{4} \gamma^\mu \gamma_\alpha \right) \left(\mathcal{R}_{\mathrm{RS}}^\alpha(\hat{\psi}_\mu) - J^\alpha \right) = 0. \tag{25}$$

Again, from the conservation of the current and the Bianchi identity for $\mathcal{R}_{\mathrm{RS}}^\mu$, taking the derivative of the equations of motion we obtain

$$\partial\!\!\!/ \left(\gamma_\alpha \mathcal{R}_{\mathrm{RS}}^\alpha(\hat{\psi}_\mu) - \gamma_\alpha J^\alpha \right) = 0. \tag{26}$$

Imposing as initial condition

$$\left(\gamma_\alpha \mathcal{R}_{\mathrm{RS}}^\alpha(\hat{\psi}_\mu) - \gamma_\alpha J^\alpha \right) \big|_0 = 0, \tag{27}$$

the last equation is equivalent to the trace of the RS case. Thus the propagator that mediates the interaction between two conserved sources is the same in both cases. In particular we find

$$\partial\!\!\!/ \hat{\psi}_{\mathrm{WRS}}^\mu = J^\mu - \frac{1}{2} \gamma^\mu \gamma_\alpha J^\alpha + \gamma^\mu \xi, \tag{28}$$

with $\partial\!\!\!/ \xi = 0$. The interaction between conserved sources can be read from the quantity

$$\bar{J}_\mu \hat{\psi}_{\mathrm{WRS}}^\mu = \bar{J}^\mu \frac{1}{\Box} \left(\eta_{\mu\nu} \partial\!\!\!/ + \frac{1}{2} \gamma_\mu \partial\!\!\!/ \gamma_\nu \right) J^\nu, \tag{29}$$

which coincides with that of the RS (see *e.g.* [15]). In particular, this guarantees the unitarity of the theory for conserved sources.

It is interesting to note that, as happens for the spin-2 Lagrangian, the WRS massive case is different from the RS one and new degrees of freedom appears in the propagator.

3.1 Remarks on the Consistent Coupling of the spin-$3/2$ field

A possible source of problems that appears in the study of theories of higher spin fields, both massive and massless, is the *consistent* coupling of the field. For the massive spin-3/2 field, it was found in [20, 21] that there are problems with unitarity and causal propagation once the field is coupled to an external electromagnetic source. For the massless case, the inconsistency of the minimal coupling of the RS field to electromagnetism occurs already at an algebraic level. Namely, if we substitute the ordinary derivative by a covariant derivative in the RS action, differentiating with the covariant derivative $D_\mu = \partial_\mu - \mathrm{i} e A_\mu$ and after using the Bianchi identity of the free RS action,

$$\partial_\mu \mathcal{R}^\mu_{\mathrm{RS}} = 0,$$

we find [11]

$$F_{\mu\nu}\gamma^\mu\psi^\nu = 0.$$

The previous expression means that either $\psi_\mu = 0$ or that the photon is a pure gauge excitation. A similar problem occurs for every massless higher spin theory, as the Bianchi identities of the free theory always imply some condition in the background field. It was suggested in [9] that the description in terms of traceless fields may alleviate this problem as the Bianchi identities are less stringent in this case.

For the WRS case, coupling minimally the action to the electromagnetic field, one finds the equations of motion

$$\left(\delta^\alpha_\mu - \frac{1}{4}\gamma^\alpha\gamma_\mu\right)\epsilon^{\mu\nu\rho\sigma}\gamma_5\gamma_\nu D_\rho\hat{\psi}_\sigma = \mathrm{i}\left(\gamma^\mu D_\mu\hat{\psi}^\alpha - \frac{1}{2}\gamma^\alpha D^\mu\hat{\psi}_\mu\right) = 0. \tag{30}$$

After applying the covariant derivative, the equations of motion read

$$eF_{\mu\nu}\gamma^\mu\hat{\psi}^\mu = \frac{\mathrm{i}}{2}\gamma^\beta D_\beta(D_\alpha\hat{\psi}^\alpha), \tag{31}$$

which is not a constraint but a field equation[9]. One can show that the previous system of equations is hyperbolic and that the propagation is causal.

The main concern about the previous coupling is that the states of low spin corresponding to $\partial_\alpha\hat{\psi}^\alpha$ are turned on by the interaction, and this may spoil the unitarity of the theory. The absence of a gauge symmetry implies that the Slavnov-Taylor identities can not be derived in the standard fashion and unitarity may be violated even at tree level. Besides, this implies that no Fadeev-Popov or Nielsen-Kallosh ghosts will be present in the quantization of the WRS action. We leave the study of these issues for future research[10] [22].

4 Supersymmetric Extensions of WTDiff

A natural question about the alternative Lagrangian describing the propagation of the spin-2 particle (WTDiff) is whether it admits a minimal supersymmetric extension. Due to the fact that

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[9]The same happens if one considers the coupling of the gauge-fixed RS action.

[10]Even if unitarity is not preserved, one could try to introduce new fields of spin-1/2 to obtain a consistent theory.

the number of *off-shell* and *on-shell* degrees of freedom of the massless WTDiff case coincides with that of the standard action for spin-2, one may wonder about the existence of an action for the spin-3/2 field such that the total *minimal* action of the WTDiff graviton plus gravitino has a certain global supersymmetry . A first sign that this may not be possible is that, as we showed in Section 3, the only Lagrangian for the field ψ_μ that describes purely spin-3/2 *on-shell* is the RS Lagrangian whose supersymmetric counterpart is the usual linearized Einstein-Hilbert action[11]. One may still think that the supersymmetric transformations can be deformed so that RS action admits another supersymmetric partner. We will study this possibility in a completely generic way.

Let us first write the form of the WTDiff action in four dimensions [1],

$$\mathcal{S}_{\text{WTDiff}} = \int \mathrm{d}^4x \left(\frac{1}{4}\partial_\mu \hat{h}^{\nu\rho}\partial^\mu \hat{h}_{\nu\rho} - \frac{1}{2}\partial_\mu \hat{h}^{\mu\rho}\partial_\nu h^\nu_\rho \right). \tag{32}$$

Under a general variation $\delta h_{\mu\nu}$ the action changes as,

$$\delta\mathcal{S}^{(2)}_{\text{WTDiff}} = \int \mathrm{d}^4x \, \delta\hat{h}_{\mu\nu} \left(R^L_{\mu\nu}(\hat{h}) - \frac{1}{2}\eta_{\mu\nu}R^L(\hat{h}) \right) =$$

$$\frac{1}{4}\int \mathrm{d}^4x \, \delta h_{\mu\nu} \left(4\eta^{ab}\eta^{\beta(\mu}\eta^{\nu)a} - 2\eta^{\alpha\beta}\eta^{a\mu}\eta^{b\nu} - \eta^{ab}\eta^{\mu\alpha}\eta^{\nu\beta} - \eta^{\mu\nu}\left\{ \eta^{\alpha a}\eta^{\beta b} - \frac{3}{4}\eta^{ab}\eta^{\alpha\beta} \right\} \right) \partial_\alpha\partial_\beta h_{ab}.$$

For the spin-3/2 Majorana field ψ_μ we will take the general action (3). The most general supersymmetric transformation for Majorana spinors and gravitons can be written as[12]

$$\delta h_{\mu\nu} = \bar{\epsilon}\gamma_{(\mu}\psi_{\nu)} + A\eta_{\mu\nu}\bar{\epsilon}\gamma^\rho\psi_\rho,$$
$$\delta\psi_\mu = \left(B\partial_\mu h + C\partial_a h^a_\mu + D\gamma_\mu\gamma^\nu\partial_\nu h + E\gamma_\mu\gamma^\alpha\partial_b h^b_\alpha + F\sigma^{ab}\partial_a h_{\mu b} \right)\epsilon. \tag{34}$$

Some of the previous transformations are simply field redefinitions or gauge transformations for certain Lagrangians but we will just consider all the coefficients as independent.

The variation of the bosonic Lagrangian can be written as

$$\delta\mathcal{S}^{(2)}_{\text{WTDiff}} = \frac{1}{4}\int \mathrm{d}^4x \, \bar{\epsilon}\Big(-\eta^{ab}\gamma^\alpha\psi^\beta + 2\eta^{\alpha a}\gamma^b\psi^\beta + 2\eta^{\alpha a}\gamma^\beta\psi^b - 2\eta^{\alpha\beta}\gamma^b\psi^a$$

$$+ \frac{3}{4}\eta^{ab}\eta^{\alpha\beta}\gamma^\rho\psi_\rho - \eta^{\alpha a}\eta^{\beta b}\gamma^\rho\psi_\rho \Big)\partial_\alpha\partial_\beta h_{ab}. \tag{35}$$

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[11]We could consider actions for the bosonic sector with more degrees of freedom *e.g.* allowing for a propagating torsion or non-metricity, but this goes beyond the present work.

[12]The supersymmetric transformation should preserve the traceless condition, which for the usual supersymmetric transformation of the graviton with $A = 0$ in (34), implies

$$\delta h = \bar{\epsilon}\gamma^\mu\psi_\mu = 0. \tag{33}$$

This seems to imply that the supersymmetric partner of the field $\hat{h}_{\mu\nu}$ should be the field $\hat{\psi}_\mu$ but, as we will see, this is not the case.

For the variation of the fermionic part we find

$$
\delta \mathcal{S}^{(3/2)} =
$$
$$
-\int \mathrm{d}^4 x \; \bar{\epsilon}\Big((2B(\lambda + \zeta) + 4D(2\lambda + \zeta) - F\lambda)\eta^{ab}\gamma^\alpha\psi^\beta + (2C\lambda + 4E(2\lambda + \zeta) + F\lambda)\eta^{\alpha a}\gamma^b\psi^\beta
$$
$$
+\zeta(2C - F)\eta^{\alpha a}\gamma^\beta\psi^b + F\zeta\eta^{\alpha\beta}\gamma^b\psi^a + (2B(\lambda + \vartheta) + 2D(\lambda + 4\vartheta - \zeta) - F\vartheta)\eta^{ab}\eta^{\beta\alpha}\gamma^\rho\psi_\rho
$$
$$
+(2C\vartheta + 2E(\lambda + 4\vartheta - \zeta) + F(\lambda + \vartheta))\eta^{\alpha a}\gamma^b\gamma^\beta\gamma^\rho\psi_\rho + \lambda(2C - F)\eta^{\alpha a}\eta^{\beta b}\gamma^\rho\psi_\rho\Big) \partial_\alpha\partial_\beta h_{ab}. \qquad (36)
$$

Comparing the third and forth coefficients of (35) and (36), we find $C = 0$. From the relation between the last coefficient and the forth one of (35), we find $\zeta = -2\lambda$. Finally, comparing the second and forth coefficient we arrive at $F\zeta = 0$, and thus there is no way in which both variations can cancel each other. Thus, we conclude that there is no minimal supersymmetric system including the WTDiff Lagrangian.

One could try to add some constraints to the action as was done in [23], to find a supersymmetric action. However, the addition of these Lagrange multipliers goes beyond the minimal coupling and can be problematic [24].

5 Conclusions

In this work, we have studied the possibility of describing the free spin-3/2 field by a Lagrangian different than the usual Rarita-Schwinger Lagrangian [13]. From the fact that the covariant fields have more components than the two physical components of the field $\pm 3/2$, we expect that the action that describes consistent spin-3/2 will be endowed with a gauge symmetry to make the extra polarizations spurious.

For the rest of the cases, we expect the theory to be *non-unitary* but we did not study the general case in detail. Instead, we identified the possible gauge symmetries and found the actions which are invariant under these transformations. It turns out that there are just two possibilities: the RS action (and all the actions related to it by a field redefinition) and a new action endowed with a S-symmetry. We have called this alternative possibility WRS, as the S-symmetry is similar to a Weyl transformation for the spin-2 field (cf. (7)) and it is related to the RS action.

For the WRS action, the study of the equations of motion reveals that, apart from the spin-3/2 polarizations, it also includes spin-1/2 degrees of freedom. Nevertheless, and as happens whenever one fixes the gauge through a covariant gauge fixing term, once coupled to conserved sources and endowed with the appropriate initial conditions, both the WRS and the RS theories give the same physical predictions. Indeed, we showed that the propagator which describes the interaction between two conserved sources coincides in both cases. In other words, both theories coincide once the extra degree of freedom is integrated out.

The introduction of interaction is always a delicate point for higher spin fields. The difficulties come from the fact that the presence of interaction typically destroys the gauge invariance of the theory. This may imply certain conditions on the background field which may render it trivial or the propagation of low spin modes which may spoil unitarity. For the usual massless spin-3/2 field, when one tries to couple it to electromagnetism, the presence of a non trivial field ψ_μ means

that the electromagnetic background must be trivial. This problem is not present in the WRS case, but, as expected, the interaction turns on the low spin modes. We leave the study of the unitarity of this coupling for future research [22].

Finally, we studied the possibility of finding a supersymmetric action built out of the WTDiff Lagrangian for spin-2. We found that the minimal possibility consisting on the addition of a certain action for a spin-3/2 field does not work. This means that the linear WTDiff action has not a (minimal) supersymmetric extension. This result is interesting because supersymmetry plays a key role in finding a consistent coupling of the spin-3/2 field [25, 11]. The previous result naively implies that gravity described from the WTDiff action can not be coupled to a spin-3/2. However, recall that the problem with the consistent coupling of the spin-3/2 is that the action should include a gauge invariance coming from a deformation of the gauge invariance of the linear action. This is the case for the coupling of the RS action to gravity where the Bianchi identities associated to the supersymmetric transformation are automatically satisfied once the Einstein's equations are imposed [11].

This seems to indicate that if one couples the spin-3/2 field in the same fashion to the WTDiff vielbein[13],

$$\hat{e}^a{}_\mu = e^{-1/4} e^a{}_\mu,$$

one will get a consistent theory once *all* the Einstein's equations are satisfied, which would mean that the integration constant that remains free in the WTDiff theory would be fixed to zero. This means that even in the absence of supersymmetry, the cosmological constant would cancel in this case. Whether this naive expectation holds or not is currently under research [22].

6   Acknowledgements

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